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ABSTRACT

This paper presents an overview of the Comprehensive School Mathematics Program (CSMP) elementary school mathematics curriculum. The philosophy behind the program, content goals, and teaching strategies are briefly discussed. Then ways in which the program develops facility in mathematics are described in discussions of the language of arrows, in which the study of relations is presented; the language of strings, with its focus on classification; and the language of the Papy minicomputer, in which computational ideas are developed. (MNS)

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PEDAGOGY AND APPROACH
OF THE
COMPREHENSIVE SCHOOL MATHEMATICS PROGRAM

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After two decades of intense activity, mathematics education continues to be an area of critical need for improvement. The work of pioneering development programs was translated into textbook series to which millions of American elementary and secondary students have been exposed. Almost from its inception, however, the reform movement came under attack and it appears today that many translations misinterpreted the letter and, more important, the spirit of the curricular reforms initiated by the early projects.

In many instances the "new" mathematics turned out to be simply "more of the same" with a frosting of "new" terminology. In others, the "new" content was applied with little or no regard to didactical analysis of the content areas or to the development of a pedagogy needed to create a good learning environment for the student. Indeed, innovations in content reforms outran reforms in pedagogy and in teacher education necessary to implement the new content successfully in the classroom. As a result the fabric of the "new math" eventually ripped and exposed a lack of thoughtful development and application of the innovations in content undertaken by the pioneering modern mathematics programs.

Notwithstanding improvements in the educational environment, concern with methods of delivering content to children and concern with precise measurement of achievement has led to a significant shift away from content-oriented programs and toward novel delivery systems. Many programs have answered the call for accountability (hence measurability) by adopting a simplistic framework of behavioral psychology. The behavioral definitions

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of knowing or understanding mathematical concepts have been shallow, even erroneous. The careful experimental analysis of mathematical behavior which could support a measurement system does not satisfy demands for immediately useful commercial programs. Thus, many program developers have opted to measure what they can measure now, rather than to discover how children exhibit their developing understanding of mathematical concepts.

Almost lost in the transition between the content-oriented programs and the accountability-conscious, delivery system-oriented programs is the fact that advances in didactics and pedagogy have occurred in mathematics education which make the content reforms of the 1960s accessible to young children. What is needed, therefore, are second-generation, content-oriented curricula whose content goals reflect those of the reform movement of the 1960s, and which also utilize the didactical and pedagogical advances. The CEMREL Mathematics and Science Group's Comprehensive School Mathematics Program (CSMP) is such a curriculum.

The philosophy of the CSMP curriculum rests on three observations, not often considered in school mathematics:

- . Mathematics is a unified body of knowledge, most profitably studied as such.
- . Mathematics involves both ideas and techniques, the former justifying and enlivening the latter.
- . Mathematics engages students most successfully in its applications, which must be chosen at a level appropriate to the students.

The content goals of the curriculum include a development of the standard number systems (natural numbers, integers, rational numbers, decimals, and real numbers) interwoven with a two-pronged development of geometry. A rich variety of experiences in combinatorial and metric geometry is presented throughout the program along with a carefully developed sequence of activities in affine (parallelogram) geometry. Moreover, the program includes a significant component of probability and statistics, beginning at the second grade and continuing throughout the program.

The unique feature of the CSMP curriculum resides in its providing an innovative and powerful pedagogical basis for meeting its content goals. The unifying mathematical ideas of sets, numbers, relations, and functions provide the framework for content development. Mathematical objects have the common property of abstractness, separate from the physical reality which they might model. This abstract, intellectual quality of mathematical objects requires that reasoning abilities be exercised in studying them. And the development of reasoning abilities is closely related to the development of imagination, ingenuity, and intuition. In response to these conditions, CSMP relies on teaching strategies which encourage creativity and allow freedom of exploration but which also foster sound intellectual habits. Learning occurs through the process of confronting a situation (real, simulated, or imagined) with genuine intellectual content, an interesting context, and a richness of consequences. As employed in the CSMP curriculum, this pedagogy of situations is demonstrably effective in:

- involving students at a personal level in studying mathematics through intriguing, stimulating contexts;
- presenting an intellectual challenge accessible to a broad range of abilities; and
- providing opportunities for individual creativity and discovery in mathematics.

Content is learned in an atmosphere of constant interrelation with applications, from simple story situations in the early grades to challenging applications in nontrivial simulations of real world problems in the upper grades. At all times the emphasis is on a dual approach to learning: understanding of content itself and its applications and, equally important, developing the techniques and processes of learning the content. The latter form of knowledge provides the power to apply the former.

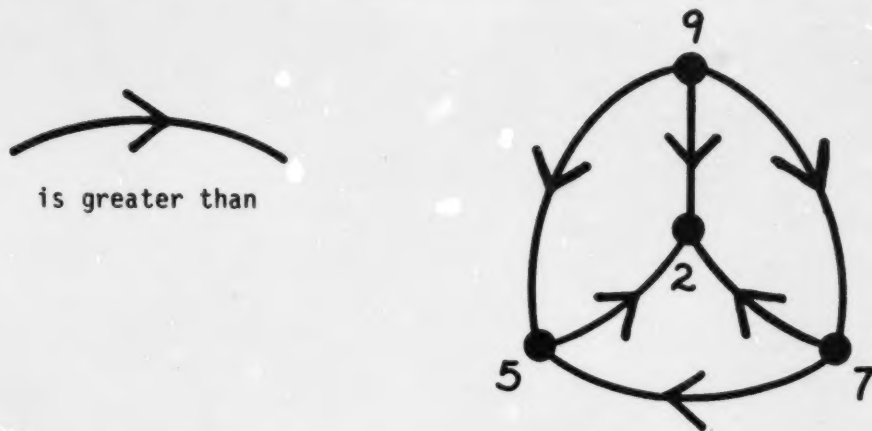
A substantive question should be directed immediately at the pedagogy. How can CSMP present situations rich in mathematical content before

students achieve verbal sophistication and computational skillfulness? A commonly held hypothesis is that students must be able to read well and to calculate accurately before approaching any, other than trivial, mathematical situations. However, the implicit additional assumption is that the student must be restricted to native, verbal languages, subject to the inherent semantical ambiguities that contribute to their power and expressiveness. To invalidate the common hypothesis by negating the hidden assumption, CSMP has developed powerful psycho-pedagogical devices which allow all students immediate access to the ideas and methods necessary not only for entering mathematical situations, but also for continually expanding their understanding of the concepts. These devices reside in three novel languages which are simple, precise, and essentially pictorial, rather than verbal. Through facility in these languages, students may develop computational skills while working in non-trivial mathematical situations. Of course, providing a viable and expressive alternative to the native languages serves the needs of other than younger students alone. Those who come to school with non-standard cultural and linguistic preparation are thereby able to enter significant mathematical study unhindered. The result is an increase in student self-esteem that may benefit the student in other areas of study.

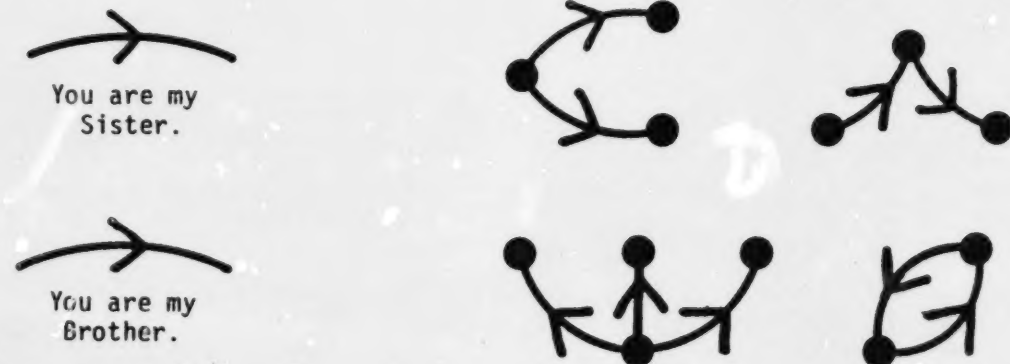
The Language of Arrows -- Relations

The study of relations among objects is a part of every intellectual pursuit, indeed, at least informally, a part of nearly every human activity. The development of mathematics can be viewed in terms of a series of discoveries of mathematical relations and their exploitation. To mention only a few: Euclid began by formalizing the relations between points and lines. Descartes developed a novel relation between pairs of numbers and points in the plane which provided an impetus for the flowering of the calculus, itself an exploitation of the relations inherent in motion. Developed with care to avoid esoteric formalism which is more properly the device of the professional mathematician, the Language of Arrows provides a compelling pictorial language for introducing relations in school mathematics. Both numerical and non-numerical relations are amenable to representation.

Consider a numerical relation such as "is greater than"; dots represent numbers in an illustration of this order relation in a collection of numbers.

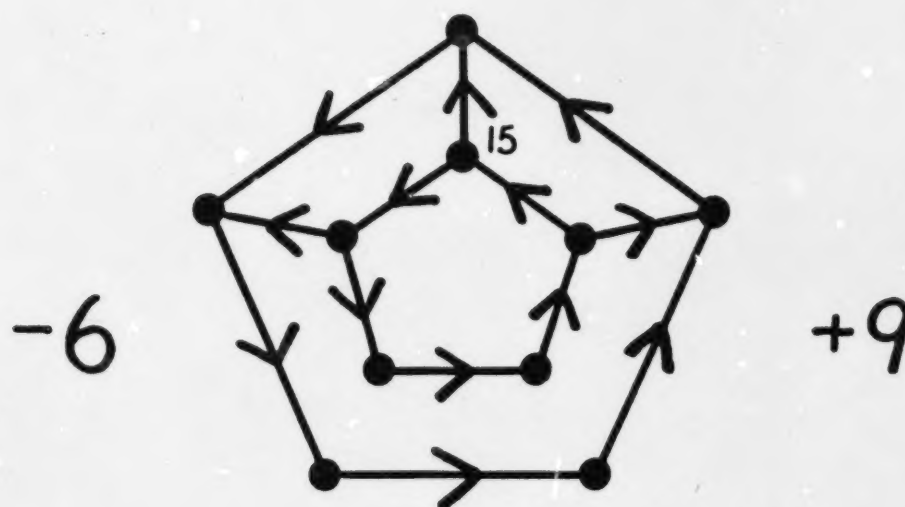


The following illustration of sibling relations employs dots to represent children.



To experience the power of the language in posing problems, the reader is invited to deduce the location of some arrows missing from the picture and even to infer the sex of some of the children (and to note any whose sex cannot be inferred from the information provided).

A powerful application of the Language of Arrows lies in representing numerical relations. In the following illustration, a red arrow signifies the function "add nine" and a blue arrow signifies "subtract six." With one number of the series identified, the students are virtually compelled to identify the remaining numbers. Calculation skills are exercised through a direct, dynamic involvement with arithmetic.



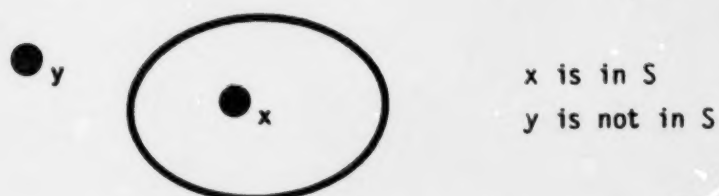
Throughout the CSMP curriculum, the Language of Arrows is used to develop the concepts of arithmetic and offer a wide range of occasions for exercising computational facility within the various number systems.

At no time is facility with the mechanisms of the language an objective, rather the development of related reasoning and computational abilities are. The danger of using arrow diagrams as simply another format for arithmetic drill requires careful attention to the development of situations. The curriculum employs situations which are thought provoking and challenging. Even very young students can work with the arrow diagram of a relation long before being able to discuss the same situation verbally. This well-constructed language frees students from the constraints of their developing verbal sophistication.

The Language of Strings -- Classification

Classifying involves arranging things in classes or sets. If the mathematics of sets has any place in an elementary school mathematics curriculum, it is precisely because it enables students to understand and to use the ideas of classification. The basis is simple: Given a set S and any object x , either x belongs to S (x is in S) or x does not belong to S (x is not in S). String pictures dramatically represent the delineation of sets by neatly illustrating the in/out sorting implicit in the definition of a set. Objects to be sorted are represented by dots and the sets into which they are to be sorted are represented by strings (see illustration).

A dot inside (the region delineated by) a set's string represents an object in the set and a dot outside a set's string represents an object not in the set.



x is in S
 y is not in S

The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity in general, and for the pursuit and understanding of mathematics in particular. In the ordinary course of events, one is not expected to deal with such problems until one's command of a native language is well developed, when one is faced with classification problems as an adult. Not that an ability to classify requires special intelligence, but it does require a means of expression. Strings and dots provide a precise, pictorial means of recording and communicating information about classification. Introduced into mathematical usage in the nineteenth century and now customarily referred to as Venn diagrams, string pictures have had a long and useful existence in assisting mathematicians and logicians to organize their information. Stripped of unnecessary formalism, the Language of Strings frees young minds to think logically and creatively about sets and to report their thinking long before they have extensive verbal skills.

In applying the Language of Strings, the drawing of dots and strings is not an objective in itself; developing relevant thinking processes is the objective. Situations must be constructed carefully with sets determined by well-defined attributes. Otherwise, students may have the distracting, peripheral problem of deciding whether a given object does or does not possess a given attribute. Thus in working with sets, CSMP typically employs attribute blocks and numbers in posing problems of classification. For example, a set of attribute blocks consists of a collection of twenty-four geometric shapes formed by taking all possible combinations of three shapes (square, circle, triangle), two sizes (large, small), and four colors (red, blue, green, yellow). Students are quickly familiar with the shapes, sizes, and colors of the attribute blocks. And the use of numerical attributes provides manifold opportunities to exercise arithmetic knowledge (for example, in testing a given number relative to the attributes "less than fifteen" or "multiple of five" or "divisor of twenty-seven").

The Language of the Papy Minicomputer -- Computation

The Papy Minicomputer, invented by the Belgian mathematician Georges Papy, is essentially an abacus which models the positional structure of our customary system of numeration. As such, it provides a powerful tool for arithmetic calculation and moreover a rich vein of situations for numerical investigations. It is not a sophisticated electronic device, but rather consists of one or more boards, each board subdivided into four squares, and a set of markers or checkers. When one or more minicomputer boards are displayed, the position that each board holds relative to the other boards corresponds to customary decimal position (ones, tens, hundreds, and so on). Often colored for pedagogical convenience as illustrated, the four squares of a board confer value on any resident checker according to the board's relative position as follows:

brown	purple
red	white

- white confers 1 (10, 100, and so on),
- red confers 2 (20, 200, and so on),
- purple confers 4 (40, 400, and so on), and
- brown confers 8 (80, 800, and so on).

The number represented by a configuration of checkers on the Minicomputer is the sum of the values of all the checkers on the boards. Of course, a number may have many different Minicomputer configurations. The Minicomputer representation of a negative number employs checkers with " $\wedge$ " written on them according to the convention that such checkers receive the negative of the value ordinarily conferred by a square. Some examples of configuration are shown below.

$$\begin{array}{|c|c|} \hline & \\ \hline & \bullet \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \\ \hline \bullet & \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \bullet \\ \hline & \\ \hline \end{array}
 \begin{array}{|c|c|} \hline \bullet & \\ \hline & \\ \hline \end{array} = 12,048$$

$$\begin{array}{|c|c|} \hline & \\ \hline \bullet & \bullet \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \bullet \\ \hline & \bullet \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array}
 \begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array}
 \begin{array}{|c|c|} \hline \bullet & \\ \hline & \bullet \\ \hline \end{array} = 35,769$$

$$\begin{array}{|c|c|} \hline & \\ \hline \bullet & \bullet \\ \hline \end{array}
 \begin{array}{|c|} \hline \\ \hline \end{array}
 \begin{array}{|c|c|} \hline \bullet & \\ \hline & \bullet \\ \hline \end{array} = 3.9
 \quad
 \begin{array}{|c|c|} \hline \bullet & \textcircled{\wedge} \\ \hline & \bullet \\ \hline \end{array}
 \begin{array}{|c|c|} \hline \bullet & \\ \hline \textcircled{\wedge} & \\ \hline \end{array} = 56$$

Used throughout the CSMP curriculum, the Minicomputer is available for a wide range of purposes and abilities. For students at an early stage of numerical development, the Minicomputer provides a means for becoming familiar even with relatively large numbers and very small numbers through the convenient representation of place values. The easy mechanics of the Minicomputer do not tax a younger student's motor skills as writing numerals often does and makes no significant demands of dexterity. But the power of the Minicomputer only becomes apparent in examining its varying role over the course of the full curriculum. In the beginning, it serves primarily as a tool for exploring numbers, their interrelationships and their anatomy through the variety of configurations available. As students grow, the Minicomputer lends itself, through an artful treatment of its mechanics, to studying a variety of algorithmic processes, a family of methods central to current advanced

mathematical thinking. In particular, standard arithmetic algorithms develop naturally from the correlative Minicomputer algorithms. As students progress, the Minicomputer is less important as a tool for calculation and more a device to stimulate mental arithmetic, to pose challenging problems about numbers, and to encourage creative thinking about the nature and properties of numbers.

Only the most narrow and elementary view of mathematics limits it to calculation. Without diminishing the importance of the number system and its uses, CSMP emphasizes the wider view that not only is intelligibility of numbers more than a matter of computational facility, but an understanding of the wider body of mathematics involves a variety of intellectual processes. The typical school mathematics curriculum emphasizes drill to develop skills in calculation. Without denying the desirability of computational skillfulness, the CSMP curriculum avoids excessive drill as such. Throughout, the essential techniques of both written and mental calculations are practiced in a natural context of engaging situations. In this way students naturally develop computational facility.